



From the history of teaching mathematics in the Middle Ages (European schools)

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ABSTRACT

The article covers the views on the teaching and development of mathematics in scientific schools in Europe in the Middle Ages. In order to cover the topic of the article, works of foreign researchers analyzing medieval sources were extensively analyzed. The conclusions drawn from the article can be used in the teaching of teachers of the future mathematics.

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O'rta asrlarda matematika o'qitish tarixidan (Yevropa maktablari)

ANNOTATSIYA

Kalit so'zlar:

matematika,
Vizantiya,
raqamlar,
Maksimus Planudes,
Misr,
Yevropa,
sxolastika,
Ibn Sino,
Sphaera materialis.

Maqolada o'rta asrlarda Yevropa ilmiy maktablarida matematika fanining o'qitilishi va rivojlanishi haqida fikr-mulohazalar yoritilgan. Maqola mavzusini yoritish maqsadida o'rta asr manbalarini tahlil qilgan xorijlik ilmiy tadqiqotchilarning asarlari keng tahlil qilindi. Maqoladan olingan xulosalarni bo'lg'usi matematika fani o'qituvchilarini o'qitishda foydalanish mumkin.

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Из истории преподавания математики в Средние века (европейские школы)

АННОТАЦИЯ

Ключевые слова:

математика,
Византия,
числа,
Максимус Планудес,
Египет,
Европа,
схоластика,
Ибн Сина,
материальная сфера.

В статье освещаются размышления о преподавании и развитии математики в научных школах средневековой Европы. В целях освещения темы статьи был проведен широкий анализ трудов зарубежных научных исследователей, анализировавших средневековые источники. Выводы из статьи могут быть использованы при обучении будущих учителей математики.

In Byzantium, before its destruction in 1453, if nothing new was done, then the old knowledge was still preserved and maintained. Of the Byzantine scientists, Maximus Planudes deserves attention, who left a book on the account, in which Arabic numerals are first found, including zero, which was called by the Hindus “figure”. Worthy of mention is also Moshopolus, who was the first to point out techniques for drawing up magic squares. Medieval Hindu mathematics had little influence on European mathematics and we will not touch on it, mentioning only three well-known Hindu mathematicians: Ariabhatta, Brahmagupta and Bhaskara Akaria. The first was predominantly an astronomer and apparently the predecessor of Copernicus, although he was born more than a thousand years earlier (in 476.).

The greatest merit of the Hindus is the system of writing numbers that later passed to us, in which the value of a digit is determined by its place (positional system). We now pass on to a strange phenomenon in history, to a people who flashed through history like a meteor and disappeared, namely, to the Arabs. The wandering people lead a solitary, nomadic life; then suddenly, under the influence of one man, Mohammed (571–632.) becomes a powerful nation and gains dominion over a significant part of the world. The first caliphs are engaged in conquests and education remains at a low level of development. But under the Abbasids (754–833) things change. Having entered into a more peaceful possession of the conquered countries, the Arabs, who at first were hostile to their culture, gradually began to show a desire to enjoy the fruits of this culture. They found Greek culture mainly in Syria. In order to benefit from Greek medicine and other Greek sciences, especially from astronomy related to astrology, they spared even the Christian and Jewish religions of those who possessed this knowledge. The Abbasids founded Baghdad in 762, and this city attained a remarkable splendor and became a center of learning; astronomy in particular enjoyed the honor that even the Mongols and Tatars, who replaced the Arabs, accorded to it.

Greek works on philosophy, natural science, astronomy and mathematics were translated. Under Al-Ma'mun and his successor Motasem, entire translation societies, Greek and Persian, were established. Thanks to this, many works have been preserved, which without this, perhaps, would not have reached us at all; for example, the work of Heron of Alexandria (on lifting weights) was considered lost for 1500 years, and only about 30 years ago were found in the Vatican Library and in the Hagia Sophia mosque in Constantinople, Arabic translations of this work and it was published. Similar academic

institutions were founded in Kufa, Damascus, Bukhara, Samarkand, etc., and the sciences and arts flourished in Persia, Egypt, North Africa, and in Europe, especially in Spain, under the caliphs Abderrahman III and Gakem II. At this time Spain, amid the dark night of barbarism, was the source of light and knowledge; in particular, the Cordoba Academy was famous, giving the world even one pope – Sylvester II (aka Herbert), who had a noticeable impact on the poor culture of Europe with his example and his writings. The Gackem library contained 600,000 manuscripts. But the Arabs did not limit themselves to only translations of Greek mathematicians; they contributed much of their own to arithmetic and trigonometry. The first outstanding mathematician of the Arabs is Mohammed ibn Muza, nicknamed Alkhvarizmi (this name, in a distorted form, became a household name and entered science under the name of Algorithm). He left the famous Algebra, mainly containing the solution of equations, especially of the 2nd degree. The title of this book “Algeber Walmukabala” was the reason that the doctrine of equations and all related operations with general quantities was called “Algebra”. Syrian Mohammed Abu Abdallah al Battani, baptized in the West in Albategnia, nicknamed the “Arab Ptolemy”, is famous for his tables of sines. He introduced the sine, i.e. a half-chord instead of a whole chord, and its name belongs to it, in Arabic jaib, which is an imitation of the Hindu “jiva” and translated in Latin by the word “sine”. He and Alkhvarismi own various works on trigonometry. The sons of a certain Muse, known as the “three brothers”, translated seven books of Apollonius on conic sections and wrote a treatise on geometry. In particular, Abul Wafa did a lot in the field of trigonometry. He compiled tables of sines and tangents and brought both flat and spherical trigonometry to a height above which it no longer rose among the Arabs. But the Arabs were engaged in trigonometry not only theoretically; they built tools and took measurements with them, among which should be mentioned the measurement of the length of the earth degree, undertaken at the command of Caliph Almamun; they built water clocks with great skill, as evidenced by the wonderful clock with various moving figures sent by Harun al-Rashid as a gift to Charlemagne. About the year 1100 the center of gravity of Arabic learning shifted to the West, to Spain, and here, in addition to the academy of Cordoba already mentioned, one can also name the higher school in Toledo; a number of Arab and Jewish scientists continued to develop algebra and trigonometry, and it can be said in conclusion that the Arabs not only preserved Greek knowledge for us, but added a lot of their own. Let's turn now to Europe. In the 7th and 8th centuries science flourished in England, especially in Ireland and Scotland. The famous English scientist Alcuin was invited by Charlemagne and he founded a school where a fairly high education is given. But it is not this court school that plays a role in the education of the people, but the monastic schools that arose in the 9th century and later. Especially famous were the schools in Fulda, St. Gallen, Reichenau, Tegernsee, Girsau, Auxerre, Cluny, Chartres, Aurillac (in France and Germany). There were even learned nuns. From the mathematical sciences, arithmetic, music and astronomy were taught, some acquaintance with which was considered necessary for future spiritual ones. Later these schools were joined by cathedral schools in Cologne, Mainz, Speyer, Konstanz, Regensburg, Augsburg, Bamberg, Laon, and Liège.

Of the scientists of this period, the most prominent is Herbert, who later became pope under the name of Sylvester II (+1003). Known for his scientific mathematical correspondence, two textbooks of arithmetic, geometry, showing familiarity with the works of Pythagoras, Plato, Eratosthenes and Heron. Great fame is enjoyed by Leonardo

Lizano, who lived in the 13th century, nicknamed Fibonacci. He traveled through Egypt, Syria, Greece, Sicily and directly acquainted with Greek and Arabic mathematics. In his great work, *Liber Abaci*, he expounds the Hindu method of reckoning, which he learned from the Arabs. He solves many problems for uncertain equations of the first degree with many unknowns, considers the approximate extraction of square and cube roots, and then equations of the 2nd degree. He also left an essay on geometry. He was close to the court of Emperor Frederick II and often at court they amused themselves by offering difficult problems, and Leonardo solved them, but this had little effect on the development of mathematics in general, and communication with the heretic emperor and his astrological situation should rather act repulsive way. Leonardo Pisano is an outstanding scientist and ahead of his time in many ways. Vitello, who lived in the 13th century, wrote *Optics*, in which he considers reflection and refraction in detail, even using conic sections. Scholasticism was little interested in mathematics, but it considered some questions of a general nature, and not without success. The Scholastics tried to define concepts as precisely as possible and naturally came to talk about continuity and infinity. They already distinguished, following the example of Aristotle, potential and actual infinity (“become infinitely large” and “be infinitely large”) or, as they said, syncategorical and categorical infinity.

During the period from 700 to 1200, teaching in the monastic and cathedral schools was kept within the framework of the quadrivium. (arithmetic, music, geometry, astronomy). From arithmetic came a little mysticism of numbers, the doctrine of proportions and practical calculations, which, depending on the age, were carried out computistically, abacistically or algorithmically (the former means calculation in the head, without any instrument, passed over from Romans, the second – calculation, with the help of abacus, special tablets with pebbles, and the third – methods of calculation that are already approaching modern ones.) In theoretical music, the doctrine of intervals was considered, geometry was passed very briefly, and in astronomy constellations, the spherical shape of the earth, the movement of the planets were radiated and time calculation. But at the beginning of the 13th century, universities appeared in Bologna, Padua, Pavia, Solerno, Salamanca, Alcalá, Coimbra, Paris, Angers, Orleans, Montpellier, Oxford, Cambridge, and then in Prague, Vienna, Heidelberg, Cologne, Leipzig, etc. However, in the universities, at first, they limited themselves only to the already existing science, not thinking of moving it forward, and the development of independent scientific thought proceeded very slowly. Of the more or less independent scientists of this epoch, we will point out the following. Of the French mathematicians, let us mention Nicholas Oresme (1323–1382). The most famous was his work “*Tractatus de latitudinibus formarum*”; In it he comes very close to the idea of Cartesian coordinates, introducing “latitude” and “longitude”, and he already shows the beginnings of analytical geometry, he already says that if one quantity is proportional the other, then you get a straight line, if not, then a curve. Jordan Nemorarius left us an essay on geometry, similar to our modern textbooks on planimetry.

Nikolai Shukle (about 1500), comparing geometric and arithmetic progressions, prepares, like Oresme, the idea of logarithms. John of Holywood, or John of Sacrobosco (+1256), left the work “*Sphaera materialis*”, which was extremely widespread and treats of spherical astronomy. John Peckham (+1292) wrote *Optics* and *Perspective*. Roger Bacon (born in 1214) possessed deep knowledge in optics and, in addition, had deep

knowledge in all sciences: mathematics, astronomy, geography, chemistry, music, medicine, 1 The remaining three sciences: grammar, rhetoric and dialectic were trivium of grammar, etc. Many of his discoveries and inventions remained unpublished (perhaps he did this for fear, as already mentioned above, of falling on the fire of the holy Inquisition), many of his works were not published, but one of his published works, *Opus Majus*, puts him in the ranks of the greatest thinkers. who wrote about science, about the causes of its stagnation and about the conditions for its progress. It is simply unbelievable that this essay was written not in the 19th, but in the 13th century! First of all, Bacon speaks of the causes of human ignorance and indicates the most important of them – the desire to hide one’s ignorance and boast of our imaginary knowledge. Then he wrestles with the authorities, and especially with Aristotle, and rightly remarks that “even the saints fell into error”. The means to the knowledge of truth are: firstly, the study of St. writings, and secondly, the study of mathematics and experience. It is truly amazing how clearly Bacon imagines the mutual relationship between mathematical deductions and the need to back them up with experiments, reinforcing this position with a number of examples. But Bacon was ahead of his time by 400, and perhaps even 600 years, since only in the 19th century did experience finally become the unshakable basis of all knowledge, and even then there were natural philosophers even in the 19th century. Peurbach (1423–1461) wrote a textbook of arithmetic, the theory of planets, trigonometry and tables of sines: the goniometric instrument proposed by him, where divisions are counted not on a circle, but on the sides of a square, deserves attention. Especially famous is Peurbach’s disciple John Müller, nicknamed Regiomontanus (1436–1476). He lived at a time when humanism had already appeared, i.e. the desire to revive the spirit of antiquity again, and when they began to study the ancient authors of Greece not from Arabic translations and alterations, but from the originals. Regiomontanus was fluent in Greek and studied the Greeks in the originals. He left tables of sines and tangents calculated with great accuracy. Regiomontanus was especially fond of trigonometry. He left an extensive work on plane and spherical trigonometry and, combining Arabic models with his own discoveries, brought trigonometry to the state in which it is taught today. In the 15th and 16th centuries enlightenment proceeds at a faster pace; urban schools appear, universities multiply, and mathematical education wins a more and more honorable place for itself. This is where we leave our brief overview of the state of mathematical knowledge in the Middle Ages.

Summarizing what has been said, we note that although individual talents appeared, such as, for example, Leonardo Pisano, Oresmus, Schuke, etc., they had little influence on the development of mathematics: the soil was still insufficiently prepared for the perception of seeds that should have been develop from these few flowers of science. The general course of the development of mathematical knowledge over the entire thousand-year period of stagnation can be compared to a slowly flowing stream; its source lies in the Greco-Arabic writings, coming especially from Spain, and then it flows through monastic schools and universities. The expansion of the circle of knowledge primarily took place in those areas in which the scholastics were interested, and the study of formal logical concepts and mechanical questions goes in an unbroken chain from Jordan to Leonardo da Vinci. But the teaching of arithmetic and geometry, which was meager at the beginning, is gradually expanding, and thanks to this, the stream perceives an influx of algebraic knowledge, the beginning of which was laid down

in the works of Leonardo Pisano. Following the practically working astronomers, university circles turned to trigonometric work, and the perspective necessary for painters posed new tasks for Geometry. The confluence of these various currents was a new starting point for new fruitful work. Thus the West, unlike the Arabs, did not appear unprepared when it was given direct access to Greek culture. This concludes our review of the state of mathematics and proceeds to mechanics. If mathematics, although to a weak degree, was nevertheless cultivated in schools and sometimes there were people who introduced something new into it, then it can be said about mechanics that during the thousand-year period of stagnation it was not studied at all and for 1000 years we have not we can point to not a single outstanding scientist and not a single work on mechanics that is in the slightest degree satisfactory. Jordanes Nemorarius left us the “Static”, but he did not go further than Aristotle. However, in the theory of the inclined plane, he has a hint of the beginning of possible displacements. Albert of Saxony says that when a heavy system is in equilibrium, the center of gravity should occupy the lowest position. These and several other similar remarks, together with a huge mass of comments (giving nothing new) by Aristotle, make up the entire literature on mechanics for 1000 years.

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